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A Method For Comparing Two Standard Deviations

Often there is a need to compare the variability of two groups or processes and ask the question 'is one process more variable than the other?'

When only two processes are to be compared, the usual method is to use the Variance Ratio test. It is run in the following way:

Define:

s_1 = the sample standard deviation of the first group,

s_2 = the sample standard deviation of the second group,

n_1 = the sample size of the first group,

n_2 = the sample size of the second group.

df_1 = the degrees of freedom of the first group, $df_1 = n_1 - 1$

df_2 = the degrees of freedom of the second group, $df_2 = n_2 - 1$

Calculate:

$$F_{calc} = \left[\left(\frac{s_1}{s_2} \right)^2 \text{ or } \left(\frac{s_2}{s_1} \right)^2 \right] \text{ whichever is larger.}$$

Lookup:

On the attached table, the value of F_{tab} . F_{tab} is obtained by finding the df for the numerator of F_{calc} in the first row and the df for the denominator of F_{calc} in the first column. The intersection of the row and column in the body of the table contains the value of F_{tab} .

Compare:

If $F_{calc} > F_{tab}$ then there is a significant difference between the two standard deviations s_1 and s_2 .

If $F_{calc} < F_{tab}$ then there is no evidence of a difference between the two standard deviations s_1 and s_2 .

An example:

$$s_1 = 2.46 \text{ and } n_1 = 7$$

$$s_2 = 5.64 \text{ and } n_2 = 12$$

Therefore:

$$df_1 = 7 - 1 = 6 \quad df_2 = 12 - 1 = 11$$

$$F_{calc} = \left[\left(\frac{2.46}{5.64} \right)^2 \text{ or } \left(\frac{5.64}{2.46} \right)^2 \right] \text{ whichever is larger} = \left(\frac{5.64}{2.46} \right)^2 = 5.26.$$

F_{tab} is obtained by finding the column headed by 11 and the row headed by 6. $F_{tab} = 4.03$

Since $5.26 > 4.03$, we can declare that the two standard deviations are significantly different (i.e. $s_2 = 5.64$ is significantly larger than $s_1 = 2.46$).

Further Considerations:

If more than two standard deviations are to be compared in pairwise fashion, it is better to use either the Brown – Forsythe or Levene procedures (both are available in JMP[®]). Both these methods require the raw data to be available. If only summary statistics are available, then Bartlett's procedure can be used but must be done by hand.

F Statistic For p = 0.05

dfn/dfd	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	22	24	26	28	30	35	40	45	50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70	8.69	8.68	8.67	8.67	8.66	8.65	8.64	8.63	8.62	8.62	8.60	8.59	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.84	5.83	5.82	5.81	5.80	5.79	5.77	5.76	5.75	5.75	5.73	5.72	5.71	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.60	4.59	4.58	4.57	4.56	4.54	4.53	4.52	4.50	4.50	4.48	4.46	4.45	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.92	3.91	3.90	3.88	3.87	3.86	3.84	3.83	3.82	3.81	3.79	3.77	3.76	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.49	3.48	3.47	3.46	3.44	3.43	3.41	3.40	3.39	3.38	3.36	3.34	3.33	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.20	3.19	3.17	3.16	3.15	3.13	3.12	3.10	3.09	3.08	3.06	3.04	3.03	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01	2.99	2.97	2.96	2.95	2.94	2.92	2.90	2.89	2.87	2.86	2.84	2.83	2.81	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.83	2.81	2.80	2.79	2.77	2.75	2.74	2.72	2.71	2.70	2.68	2.66	2.65	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.69	2.67	2.66	2.65	2.63	2.61	2.59	2.58	2.57	2.55	2.53	2.52	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.57	2.56	2.54	2.52	2.51	2.49	2.48	2.47	2.44	2.43	2.41	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.51	2.50	2.48	2.47	2.46	2.44	2.42	2.41	2.39	2.38	2.36	2.34	2.33	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.44	2.43	2.41	2.40	2.39	2.37	2.35	2.33	2.32	2.31	2.28	2.27	2.25	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.38	2.37	2.35	2.34	2.33	2.31	2.29	2.27	2.26	2.25	2.22	2.20	2.19	2.18
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.33	2.32	2.30	2.29	2.28	2.25	2.24	2.22	2.21	2.19	2.17	2.15	2.14	2.12
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.29	2.27	2.26	2.24	2.23	2.21	2.19	2.17	2.16	2.15	2.12	2.10	2.09	2.08
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.25	2.23	2.22	2.20	2.19	2.17	2.15	2.13	2.12	2.11	2.08	2.06	2.05	2.04
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.18	2.17	2.16	2.13	2.11	2.10	2.08	2.07	2.05	2.03	2.01	2.00
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.23	2.20	2.18	2.17	2.15	2.14	2.12	2.10	2.08	2.07	2.05	2.04	2.01	1.99	1.98	1.97
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.13	2.11	2.10	2.08	2.07	2.05	2.03	2.01	2.00	1.98	1.96	1.94	1.92	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.00	1.98	1.97	1.95	1.94	1.91	1.89	1.88	1.86
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	2.05	2.03	2.02	2.00	1.99	1.97	1.95	1.93	1.91	1.90	1.87	1.85	1.84	1.82
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.93	1.91	1.90	1.88	1.87	1.84	1.82	1.80	1.79
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.99	1.98	1.96	1.95	1.93	1.91	1.89	1.87	1.85	1.84	1.81	1.79	1.77	1.76
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.08	2.04	2.01	1.99	1.96	1.94	1.92	1.91	1.89	1.88	1.85	1.83	1.82	1.80	1.79	1.76	1.74	1.72	1.70
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.85	1.84	1.81	1.79	1.77	1.76	1.74	1.72	1.69	1.67	1.66
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05	2.01	1.97	1.94	1.92	1.89	1.87	1.86	1.84	1.82	1.81	1.78	1.76	1.74	1.73	1.71	1.68	1.66	1.64	1.63
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95	1.92	1.89	1.87	1.85	1.83	1.81	1.80	1.78	1.76	1.74	1.72	1.70	1.69	1.66	1.63	1.61	1.60
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.76	1.75	1.72	1.70	1.68	1.66	1.65	1.62	1.59	1.57	1.56
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.93	1.89	1.86	1.84	1.81	1.79	1.77	1.75	1.74	1.72	1.70	1.67	1.65	1.64	1.62	1.59	1.57	1.55	1.53
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.91	1.88	1.84	1.82	1.79	1.77	1.75	1.73	1.72	1.70	1.68	1.65	1.63	1.62	1.60	1.57	1.54	1.52	1.51
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85	1.82	1.79	1.77	1.75	1.73	1.71	1.69	1.68	1.65	1.63	1.61	1.59	1.57	1.54	1.52	1.49	1.48
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80	1.77	1.74	1.72	1.69	1.67	1.66	1.64	1.62	1.60	1.57	1.55	1.53	1.52	1.48	1.46	1.43	1.41
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.81	1.77	1.74	1.71	1.69	1.66	1.64	1.62	1.61	1.59	1.56	1.54	1.52	1.50	1.48	1.45	1.42	1.40	1.38
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.80	1.76	1.73	1.70	1.68	1.65	1.63	1.61	1.60	1.58	1.55	1.53	1.51	1.49	1.47	1.43	1.41	1.38	1.36
>1000	3.84	3.00	2.61	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75	1.72	1.69	1.67	1.64	1.62	1.61	1.59	1.57	1.54	1.52	1.50	1.48	1.46	1.42	1.40	1.37	1.35

dfn = degrees of freedom for numerator dfd = degrees of freedom for denominator