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Using Bartlett's Procedure for Determining the Equality of Several Standard Deviations.

Given –

a series of p standard deviations: s_i i = 1,….., q

the corresponding sample sizes for each standard deviation: n_i i = 1,…..,q.

Calculate –

the pooled variance $$S_p^2 = \frac{\sum_{i=1}^{q}(n_i - 1)s_i^2}{\sum_{i=1}^{q}(n_i - 1)}$$

Bartlett's statistic $$\mathbf{B} = \ln(S_p^2)\sum_{i=1}^{q}(n_i - 1) - \sum_{i=1}^{q}(n_i - 1)\ln(s_i^2).$$

Compare –

Bartlett's B to the 99th percentile of the χ^2 distribution with q-1 degrees of freedom. If B is less than the χ^2 critical value, then the standard deviations are homogeneous (equal). Otherwise, at least one of the standard deviations is not equal to the others.

Example –

Consider the data in the following table.:

Sample	Standard Deviation	Sample Size
1	29.1	168
2	28.9	167
3	30.1	144
4	35.1	161
5	38.6	151
6	52.1	44

Bartlett's calculation would be:

$$S_p^2 = \frac{167 * 29.1^2 + 166 * 28.9^2 + 143 * 30.1^2 + 160 * 35.1^2 + 150 * 38.6^2 + 44 * 52.1^2}{167 + 166 + 143 + 160 + 150 + 44} = \mathbf{1144}$$

$$\begin{aligned} B = \ln(1144) * [167 + 166 + 143 + 160 + 150 + 43] \\ - 167 * \ln(29.1) - 166 * \ln(28.9) - 143 * \ln(30.1) - 160 * \ln(35.1) - 150 * \ln(38.6) - 43 * \ln(52.1) \end{aligned}$$

$\mathbf{B = 45.9}$ with **df = 5**.

The χ^2 critical value is 15.1 at the 99th percentile.

Since **B** exceeds the χ^2 critical value, declare that at least one of the standard deviations is different than the others.

The next step would be to exclude extreme standard deviations from the data set and re-compute Bartlett's test to determine which ones are different from the rest.

Bartlett's test is particularly useful when only the summary data are available (standard deviation and sample size). If the raw data are available, the Brown-Forsythe or Levine's Test are preferred.